Steady Unidirectional Flow

Steady flow implies that no change occur with time. Flow condition differ for confined and unconfined aquifer and hence used to be consider separately, beginning with flow in one direction.

Steady flow in Confined Aquifer :

If there is a steady movement of groundwater in a confined aquifer, there will be a linear gradient or slope to the piezometric surface; i.e., its two-directional projection is a straight line. For this type of groundwater flow, Darcy's law can be directly applied. In Fig. 1.1, a portion of a homogeneous and isotropic confined aquifer of uniform thickness is shown wherein the hydraulic head has a linear gradient. Two observation wells/piezometers are installed L distance apart in the aquifer where the hydraulic head can be measured.

Using Darcy's law, the quantity of groundwater flow per unit width of the aquifer (q) can be determined as:

$$q = Kb\frac{dh}{dx} \qquad (1.1)$$

Where, K = mean hydraulic conductivity of the confined aquifer, b = thickness of the confined aquifer, and $\frac{dh}{dx}$ = hydraulic gradient in the X-direction.

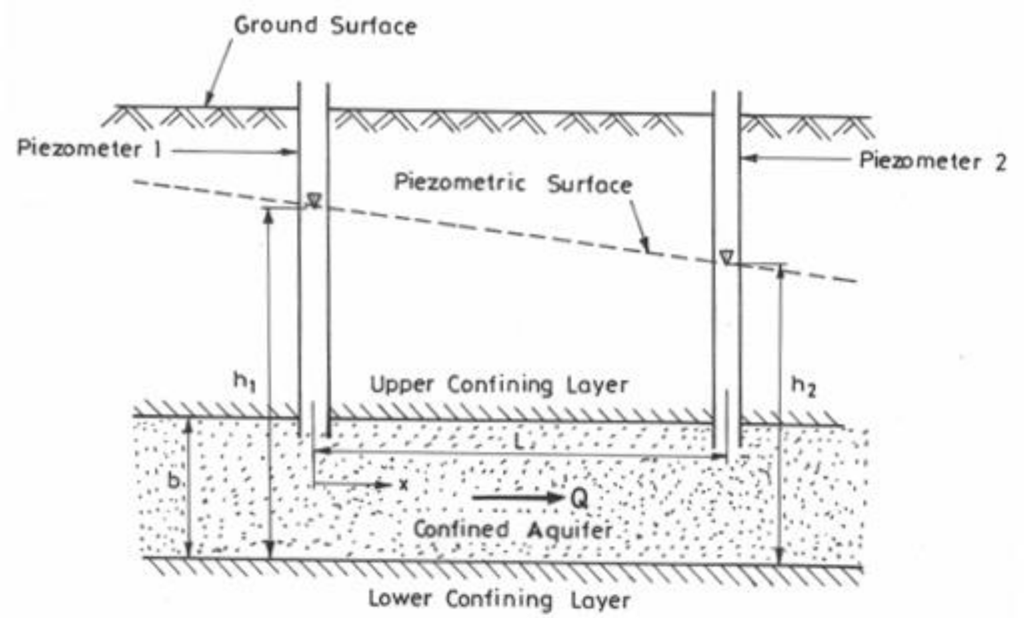

Fig. 1.1. Steady flow through a confined aquifer of uniform thickness.

(Modified from Fetter, 1994)

One may be interested to know the hydraulic head (h) at some intermediate distance, x between Piezometer 1 having hydraulic head h_1 and Piezometer 2 having hydraulic head h_2. This can be determined from the following equation:

$$h(x) = h_1 - \frac{q}{Kb}x \qquad (1.2)$$

Where, h(x) = hydraulic head at distance x, and x = distance from Piezometer 1.

Steady Flow in Unconfined Aquifers

In unconfined aquifers, as illustrated in Fig. 1.2, the fact that the water table constitutes the upper boundary of the groundwater flow region complicates flow determination. The shape of the water table determines the flow distribution, but at the same time the flow distribution governs the water-table shape. Therefore, a direct analytical solution of the Laplace equation is not possible in this case.

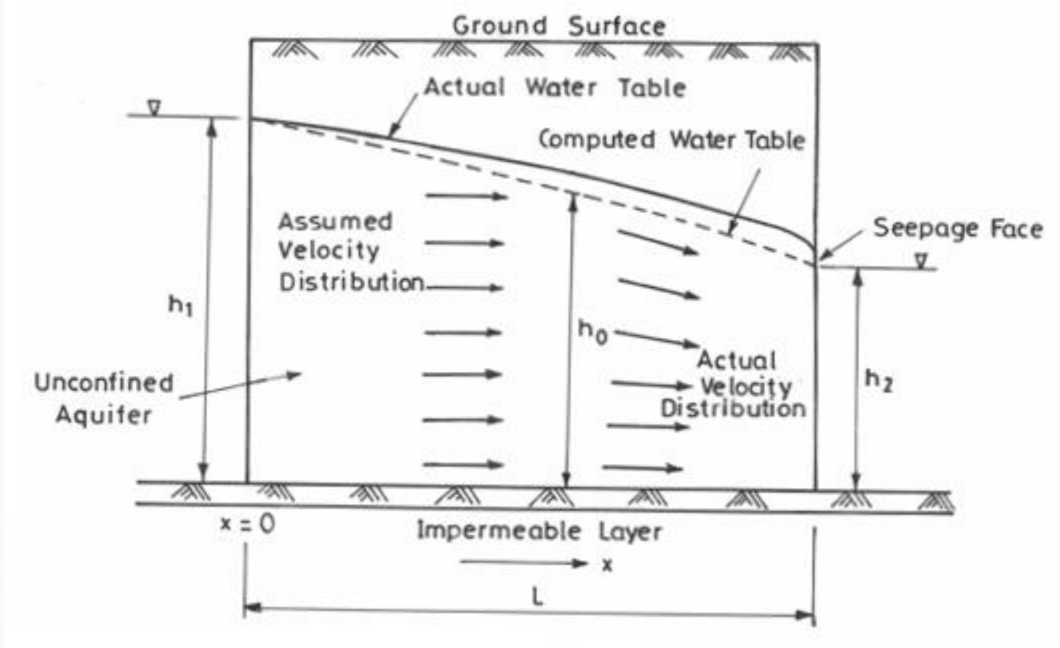

Fig. 1.2. Steady flow in an unconfined aquifer between two water bodies with vertical boundaries. (Modified from Todd, 1980)

For similar flow situation in an unconfined aquifer direct analytic solution of the Laplace equation is not possible. The difficult arises from the fact that the water in two dimensional case represents flow line. The shape of the water table determines the flow distribution, but at the same time the flow distribution governs the water table shape. To obtain a solution Dupuit assume:

1. The velocity of the flow to be proportional to the tangent of the hydraulic gradient .
   $V = -K\, dh/ds$
2. The flow to be horizontal and uniform everywhere in a vertical section.
   These assumption although permitting a solution to be obtained, limit the application of the result.
   For unidirectional flow
   $h = C_1 x + C_2$ (C_1 and $C_2$are constant of intergration)…………… (1)
   the discharge per unit width q at any vertical section can be given as
   $q = - Kh\, dh/dx$ (2)
   where k is hydraulic conductivity,
   h is the height of the water table above an impervious base and X is the direction of flow

$q = -kh/2 + C$ (3)

and if $h = h_0$ where $X = 0$ then the Dupuit equation:
$q = K/2X \ (h_0^2 - h^2)$ (4)
results, which indicates that the water table is parabolic in form.
For flow between two fixed bodies of water of constant head h_0 and h_1 the water the water table shape at the upstream boundary of the aquifer
$dh/dx = -q/Kh_0$ (5)
But the boundary $h = h_0$ is an equipotential line because the fluid potential in a water body is constant, consequently , water table must be horizontal which is inconsistent . In the direction flow, the parabolic water described by equation no. 4 increases in slope by doing so the two dupuit assumption previously stated become increasingly poor approximation to the actual flow , therefore the actual water table deviates more and more from the computed position in the direction of flow. The fact that the actual water table lie above the computed one can be explained by the fact that Dupuit flow are all assumed horizontal, whereas the actual velocities of the same magnitude have a downward vertical component so that a greater saturated thickness is required for the same discharge. At the downstream boundary a discontinuity in flow form because no consistent flow pattern can connected a water table directly to a downstream force water surface. The water table actually approaches the boundary tangentially above the water body surface and from seepage face.

Sample Question :

1. Discuss the Groundwater flow under steady condition.

   Or

2. Write a notes on steady flow in :
   i. Steady flow in confined aquifer
   ii. Steady flow in unconfined aquifer.